A New Technique for Measuring an Electromagnetic Field by a Coil Spring*

KEIGO IIZUKA†, MEMBER, IEEE

Summary—A thin coil spring whose length can be varied periodically by mechanical means has been used as a probe to measure the electric field intensity. The vibrating probe acts as a reradiating antenna with periodically varying length that modulates and scatters the component of the $\bar{E}$ field which is parallel to the axis of the spring. The scattered signal is picked up by a receiving antenna (the sending antenna was used also as a receiving antenna) and amplified with an amplifier that is locked-in with the frequency of the mechanical vibration of the coil.

Measurements were made with coil springs of three different dimensions, and a gain constant K, which should be independent of the product of the axial length of the coil and the magnitude of the vibration, was experimentally checked.

Measurements of the field of a half-wave dipole antenna as measured with the spring probe are in general agreement with theory.

An expression for the ratio between the scattered and incident waves is obtained based upon the field-pattern method and the approximations involved in the reciprocity-theorem method are clarified. It is also demonstrated (in the Appendix) that the effective length of a thin helical antenna with a triangular current distribution equals one-half of the axial length of the helix.

List of Symbols

α = helix angle whose axis oriented along the z-axis

β = propagation constant of free space $2\pi/\lambda$

β' = phase constant of the current along the wire of the helix defined as $I=I_0e^{-j\beta'\phi}$

ζ_0 = intrinsic impedance of free space

θ_1 = angle between the axis of the antenna (1) and the line to the center of the coil spring probe (2)

θ_2 = angle between the axis of the coil spring probe (2) and the line to the center of the antenna (1)

λ = wavelength in free space

ν = frequency of the axial vibration of the coil spring

ϕ = spherical coordinate (azimuth)

ψ = angle orienting the coil spring probe (2) with respect to the antenna (1) (see Fig. 1)

ω_c = angular frequency of the RF signal

a = radius of the helix

D = diameter of the coil spring

$\bar{E}_{21}(R_0, \theta_1, \phi_1)$ = electric field intensity at (R_0, θ_1, ϕ_1) (or at the probe) set by $I_1(0)$

$E_{21R}(R_0, \theta_1)$ = R component of $\bar{E}_{21}(R_0, \theta_1, \phi_1)$

$E_{21\theta}(R_0, \theta_1)$ = θ component of $\bar{E}_{21}(R_0, \theta_1, \phi_1)$

$E_{21\phi}(R_0, \theta_1)$ = ϕ component of $\bar{E}_{21}(R_0, \theta_1, \phi_1)$

$\bar{E}_{12}(R_0, \theta_2, \phi_2)$ = electric field intensity at (R_0, θ_2, ϕ_2) (or at the antenna under test) set by $I_{21}(0)$

$E_{12\theta}(R_0, \theta_2)$ = θ component of $\bar{E}_{12}(R_0, \theta_2, \phi_2)$

$E_{12\phi}(R_0, \theta_2)$ = ϕ component of $\bar{E}_{12}(R_0, \theta_2, \phi_2)$

$\bar{E}$ = electric field set up by the helix of length h_2 and radius a

E_θ = θ component of $\bar{E}$

ΔE_θ = deviation in E_θ due to the increment in h_2

E_ϕ = ϕ component of $\bar{E}$

ΔE_ϕ = deviation in E_ϕ due to the increment in h_2

E_ϵ = $\bar{E}$ field component tangential to the ellipsoid

e_c = eccentricity of the ellipsoid

e_h = eccentricity of the hyperboloid

$F_{10}(\theta_1, \beta h_1)$ = field factor of the dipole antenna of length h_1

$F_{20}(\theta_2, \beta h_2)$ = field factor of the coil spring of length h_2

h_2 = physical axial length of the coil spring

h_{e2} = effective length of the coil spring, $h_{e2} \doteqdot h_2/2$ when $h_2 \ll \lambda$

Δh_2 = increment in h_2

Δh_{e2} = magnitude of the increment in h_{e2} in the case of vibration

$\Delta h_{e2}(t)$ = instantaneous value of the increment in h_{e2}

I = current on the helix with a triangular distribution decreasing toward the ends of the helix (coil spring)

I_0 = value of the current I at the center of the coil spring

$I_1(0)$ = value of the current $I_1(z)$ on the antenna (1) at the terminal $z=0$

$I_{21}(0)$ = current induced at the load terminal of scatterer by $E_{21\theta}(R_0, \theta_1)$

$I_{12}(0)$ = current induced at the driving-point of the antenna (1) by $E_{12\theta}(R_0, \theta_2)$

$\Delta I_{12}(0)$ = deviation in $I_{12}(0)$ due to the increment in h_2

K = gain constant of the system $(\Delta I_{12}(0)/h_{e2}\cdot\Delta h_{e2})$; the quantity which is constant regardless of the properties of the coil spring

$K_0 = Z_1/V_1$

m = modulation factor, ratio of a departure of the modulation envelope from the carrier level to the carrier level

n = number of turns of the helix

$p = \beta a \sin\theta$

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† Gordon McKay Lab., Harvard University, Cambridge, Mass.

$q = \beta a \tan \alpha \cos \theta$

R = distance between the point P and an arbitrary point on the helix identified by the angle ϕ

R_0 = distance between the point P and the center of the helix

$\hat{s}$ = unit vector along the wire of the helix

V_1 = potential across the driving-point of antenna (1) with the load on the antenna terminals

Z_{01} = driving-point impedance of the antenna (1)

Z_{L1} = impedance of the load of the antenna (1)

Z_{02} = driving-point impedance of the coil spring

Z_{L2} = impedance of the load of the coil spring ($Z_{L2} = 0$)

Introduction

In practice, the probe for measuring the electromagnetic field intensity usually can be reduced sufficiently in size so that its disturbing effect on the field to be measured may be ignored. On the other hand, the disturbing effects of the lead wires to the probe are often hard to reduce to negligible proportions except in special cases when a plane of symmetry permits the use of an image-plane method. Hence, the attempt has been made by several authors to reduce the effect of the lead wires in one way or another, and each has advantages and disadvantages. Harries,[1] Hansen,[2] Maier,[3] Maier and Slater[4,5] and Shefer[6] have measured the local electric and magnetic fields in an enclosed resonant cavity by inserting a metal sphere, needle or disk of dimensions small compared to a wavelength. The field intensity can be determined by the shift of the resonant frequency of the cavity. This method, however, is limited to closed systems. Casimir[7] was the first to propose the possibility of measuring the field strength in the aperture of a horn which is connected to a cavity by measuring the difference in the standing-wave patterns in the cavity with and without a scattering element in the horn. Justice and Rumsey[8] also used the same principle and tried to separate the echo from the scatterer by means of a hybrid junction. The difficulties with the last two methods are that either the difference in the standing-wave pattern or the magnitude of the echo signal is so small that these quantities are easily overshadowed by a slight drift in the source frequency or by a mechanical imperfection in the hybrid junction unless a very large echo signal is obtained by placing the scattering element very close to the aperture. Richmond[9] and Strait and Cheng[10] devised a scatterer consisting of a dipole and a loop loaded with a germanium diode which can amplitude modulate its back-scattering cross section by applying the ac bias voltage at an audio frequency through thin, poorly-conducting wires made of cotton thread dipped into Aquadag. This method has the advantage of easy separation of the reflected signal from the incident wave. It is also characterized by a high signal-to-noise ratio in the detected amplitude modulated signal owing to the utilization of a narrow-band lock-in amplifier. Possible disadvantages are the presence of the poorly-conducting lead wire which supplies the bias-voltage across the germanium diode, and the ease with which stray noise may be picked up by the lead wires owing to the high input impedance to the diode. Cullen and Paar[11] also amplitude modulated the scattered signal by rotating a thin scattering dipole at a constant speed about an axis perpendicular to its length so that the reflected waves fluctuated in a characteristic manner. In this way no conducting lead wires are exposed to the radiation. However, since a nylon cord on whose center a dipole is mounted was kept taut and was spun by a motor, the entire probe system was quite bulky and not very maneuverable. This method does not permit the measurement of fields other than those that are linearly polarized without a previous knowledge of the polarization of the field with respect to the orientation of the probe. Scharfman and King[12] measured the radar cross section of the object by modulating the position of the object. Iizuka[13,14] determined the distribution of an $\overline{E}$ field (or $\overline{H}$ field) by measuring an amplitude modulated signal reflected from a small dipole (or shielded loop) which is loaded with a photocell that is illuminated by a chopped light beam. Iizuka,[15] recently built a minia-

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[3] L. C. Maier, "Field Strength Measurements in Resonant Cavities," Massachusetts Institute of Technology, Cambridge, Mass., Rept. No. 143; 1949.

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[8] R. Justice, and V. H. Rumsey, "Measurement of electric field distributions," IRE Trans. on Antennas and Propagation, vol. AP-3, pp. 177–180; October, 1955.

[9] J. H. Richmond, "A modulated scattering technique for measurement of field distributions," IRE Trans. on Microwave Theory and Techniques, vol. MTT-3, pp. 13–15; July, 1955.

[10] B. J. Strait, and D. K. Cheng, "Microwave magnetic-field measurements by a modulated scattering technique," *Proc. IEE*, vol. 109B, pp. 33–39; January, 1962.

[11] A. L. Cullen, and J. C. Parr, "A new perturbation method for measuring microwave field in free space," *Proc. IEE*, vol. 102B, pp. 836–844; November, 1955.

[12] H. Scharfman, and D. D. King, "Antenna-scattering measurements by modulation of the scatterer," Proc. IRE, vol. 42, pp. 854–858; May, 1954.

[13] K. Iizuka, "How to measure field pattern with photo-sensitive probes," *Electronics*, vol. 36, pp. 39–43; January 25, 1963.

[14] K. Iizuka, "A photo-conductive probe for measuring electromagnetic fields," to be published by *Proc. IEE*.

[15] K. Iizuka, "Leadless transceiver probe works underwater," *Electronics*, vol. 36, pp. 56–59; July 19, 1963.

ture FM wireless transceiver for measuring the field intensity. Both of Iizuka's methods completely eliminate the use of either mechanical or electrical connections to the probe.

A Method of Measuring Field Strength by Using a Coil Spring as a Probe

It is possible to determine the distribution of the field strength of an electromagnetic wave by measuring the modulated signal reflected from a coil spring, the axial length of which is varied periodically.

Fig. 1 shows a geometrical configuration of an antenna ① whose radiation pattern is to be studied and a coil-spring scatterer ② used as a probe. The primary radiated field $E_{21\theta}(R_0, \theta_1)$ from the antenna ① is partially reflected by the coil-spring scatterer ②. This secondary reflected signal $E_{12\theta}(R_0, \theta_2)$ (which is proportional to the component of the primary field $E_{21\theta}(R_0, \theta_1)$ which is parallel to the axis of the spring) is received by the antenna ①.

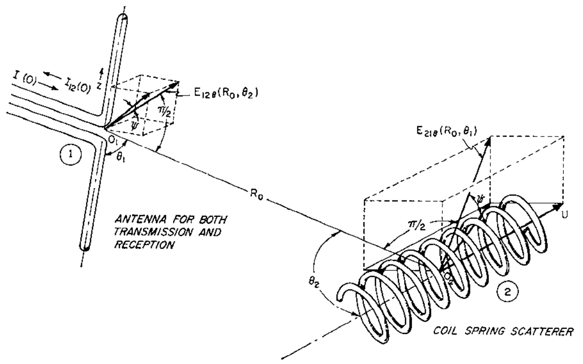

Fig. 1—Geometrical configuration of the antenna whose radiation pattern is under investigation and coil spring used as a scatterer.

A general expression for the back-scattered signal V from a thin loaded dipole has been derived by a number of authors using a reciprocity theorem.[8,11,16] All their results are equivalent to

$$V = V_0 - K_0 \frac{E_{21}{}^2 h_e{}^2}{Z + Z_L},$$

where Z, Z_L, h_e, E_{21} and V_0 are driving-point impedance, load impedance, effective length of the probe, the field intensity at the probe and the back-scattered signal from an open-circuited dipole probe which is implicitly a function of h_e, respectively. V is a reflected signal (voltage) on the transmission line. $K_0 = Z_1/V_1$ where Z_1 and V_1 are driving-point impedance and the potential across the driving point of the antenna ①. It might be worthwhile here to derive an expression which corresponds to the one above in another manner in which V_0 is explicitly a function of h_e, and which is perhaps somewhat more physical and clarifies the obscurities involved in deriving the above equation. It is assumed that the coil spring is placed in the region where the condition

$$R_0 \gg h_1 \quad \text{or} \quad h_2 \tag{1}$$

is satisfied. The treatment for the region closer to the antenna than this can be made in a similar way by simply adding extra $1/R^2$ terms.

Let the origin of a coordinate system R_1, θ_1, ϕ_1 be located at the center of antenna ①. Let a second system of coordinates R_2, θ_2, ϕ_2 be located at the center of the coil spring ②. The field $\overline{E}_{21}(R_0, \theta_1, \phi_1)$ set up by a dipole antenna ① of half-length h_1 at a distance R_0 from the current $I_1(0)$, consists of the components[17]

$$E_{21R}(R_0, \theta_1) = \frac{I_1(0)\zeta_0 h_1}{2\pi R_0} \frac{e^{-j\beta R_0}}{R_0} \sin(\beta h_1 \cos\theta_1)$$

$$(\doteq 0 \text{ if } R_0 \gg h_1)$$

$$E_{21\theta}(R_0, \theta_1) = \frac{jI_1(0)\zeta_0}{2\pi} \frac{e^{-j\beta R_0}}{R_0} F_{10}(\theta_1, \beta h_1) \tag{2}$$

$$E_{21\phi}(R_0, \theta_1) \doteq 0,$$

where

$$F_{10}(\theta_1, \beta h_1) = \frac{\cos(\beta h_1 \cos\theta_1) - \cos\beta h_1}{\sin\theta_1 \sin\beta h_1}.$$

$$\zeta_0 = \sqrt{\frac{\mu}{\epsilon}}.$$

Due to the rotational symmetry with respect to ϕ, $E_{21\theta}(R_0, \theta_1, \phi_1)$ is abbreviated by $E_{21\theta}(R_0, \theta_1)$. $F_{10}(\theta_1, \beta h_1)$ is a complex field factor for transmission by antenna ①. Note that $E_{21\theta}(R_0, \theta_1)$ is the only component which is nonvanishing in the region $R_0 \gg h_1$. The current induced at the load terminal of the scatterer by $E_{21\theta}(R_0, \theta_1)$ is

$$I_{21}(0) = \frac{2h_{e2}(\theta_2)E_{21\theta}(R_0, \theta_1)\cos\psi}{Z_{02} + Z_{L2}}, \tag{3}$$

where ψ is an angle orienting the coil spring ② with respect to the direction of the polarization of $E_{21\theta}(R_0, \theta_1)$ (see Fig. 1). $h_{e2}(\theta_2)$ is the complex effective length of a scatterer when used for reception. Effective length $h_{e2}(\theta_2)$ is usually defined as a function of the direction of the incident wave and the length of the antenna. (See the literature for detailed discussions.[17]) Z_{02} and Z_{L2} are the driving-point impedance and load impedance of the scatterer, respectively; in this case, $Z_{L2} = 0$.

The current $I_{21}(0)$ through the load terminals of the scatterer sets up the field $E_{12\theta}(R_0, \theta_2)$ at the antenna ①,

$$E_{12\theta}(R_0, \theta_2) = j\frac{I_{21}(0)\zeta_0}{2\pi} \frac{e^{-j\beta R_0}}{R_0} F_{20}(\theta_2, \beta h_2). \tag{4}$$

[16] M. K. Hu, "On measurements of microwave E and H field distribution by using modulated scattering methods," IRE Trans. on Microwave Theory and Techniques, vol. MTT-8, pp. 295–300; May, 1960.

[17] R. W. P. King, "The Theory of Linear Antennas," Harvard University Press, Cambridge, Mass., pp. 568–571; 1956.

It has not been mentioned in the literature that $F_{20}(\theta_2, \beta h_2)$ is the complex field factor for transmission by the scatterer with the receiving distribution of current which is not, in general, the same as the distribution along a center-driven antenna.[17] It follows that, in general, $F_{20}(\theta_2, \beta h_2)$ is not equal to $\beta h_{e2}(\theta_2)$. Therefore, the application of the reciprocity theorem to a driven and reradiating structure is not strictly rigorous if it implies the assumption that the field pattern of the reradiating antenna is the same as the field pattern of the same antenna when driven by a voltage across a pair of terminals. In a similar way, the current induced on the antenna ① due to the field $E_{12\theta}(R_0, \theta_2)$ is

$$I_{12}(0) = \frac{2h_{e1}(\theta_1)E_{12\theta}(R_0, \theta_2)\cos\psi}{Z_{01} + Z_{L1}}, \tag{5}$$

where Z_{01} and Z_{L1} are the driving-point and load impedance of the antenna ①. Inserting (2–4) into (5),

$$I_{12}(0) = -\frac{I_1(0)}{\pi^2}\left(\frac{\zeta_0 e^{-j\beta R_0}}{R_0}\right)^2 \cdot \frac{F_{10}^2(\theta_1, \beta h_1)h_{e2}(\theta_2)F_{20}(\theta_2, \beta h_2)}{(Z_{01} + Z_{L1})(Z_{02} + Z_{L2})\beta}\cos^2\psi, \tag{6}$$

where the relation $\beta h_{e1}(\theta_1) = F_{10}(\theta_1, \beta h_1)$ is used, since $F_{10}(\theta_1, \beta h_1)$ is the field pattern of antenna ① when center driven. Note that if the approximation

$$F_{20}(\theta_2, \beta h_2) \doteq \beta h_{e2}(\theta_2)$$

is a good one (as in the case of electrically short antennas), then (6) reduces to a form similar to (1):

$$\frac{I_{12}(0)}{I_1(0)} = -\frac{1}{\pi^2}\left(\frac{\zeta_0 e^{-j\beta R_0}}{R_0}\right)^2 \cdot \frac{F_{10}^2(\theta_1, \beta h_1)h_{e2}^2(\theta_2)}{(Z_0 + Z_{L1})(Z_0 + Z_{L2})}\cos^2\psi. \tag{7}$$

The preceding analysis was carried out specifically for the far-zone field pattern and effective length, but the same procedure can be applied to more general fields.

In order to facilitate the separation of the scattered signal from any small part of the incident wave that may leak through the hybrid junction or may have been reflected from the elements that support the coil spring, the scattered signal from the probe was amplitude modulated by mechanically vibrating the coil spring in its axial direction. It is demonstrated in the Appendix that the effective length of a thin helix is approximately equal to one-half of the physical axial length of the helix (see (25)) if the current distribution along the helix is assumed to be a triangular shape.

Eq. (7) shows that the scattered signal $I_{12}(0)$ is proportional to the square of the effective length, hence the departure $\Delta I_{12}(0)$ of the modulation envelope from the carrier level of the scattered wave due to a displacement, $\Delta h_{e2}(t) = \Delta h_{e2}\sin(2\pi\nu t + \theta)$, of the effective length is,

$$\Delta I_{12}(0) \propto (h_{e2} + \Delta h_{e2}(t))^2 - h_{e2}^2 = 2\Delta h_{e2}(t)\cdot h_{e2} + (\Delta h_{e2}(t))^2.$$

Note, the last term does not give any contributions regardless of the magnitude of $\Delta h_{e2}(t)$.

$$(\Delta h_{e2}(t))^2 = \frac{(\Delta h_{e2})^2}{2}(1 - \cos 2(2\pi\nu t + \theta)).$$

This adds a constant and a double frequency term, but the term in ν is not changed. The magnitude of $\Delta h_{e2}(t)$, however, is restricted to $|\Delta h_{e2}/h_{e2}| \ll 1$ due to the fact that beyond this range the effective length of a thin helix can no longer be considered as a linear function of its physical axial length as is discussed in the Appendix.

The complete expression for $\Delta I_{12}(0)$ is,

$$\Delta I_{12}(0) = \frac{-1}{\pi^2} I_1(0)\left(\frac{\zeta_0 e^{-j\beta R_0}}{R_0}\right)^2 \frac{F_{10}^2(\theta_1 \beta h_1)h_{e2}(\theta_2)}{(Z_0 + Z_{L1})(Z_0 + Z_{L2})} \cdot(\cos^2\psi)2\Delta h_{e2}\sin(2\pi\nu t + \theta), \tag{8}$$

with the modulation factor m given by

$$m = \frac{2\Delta h_{e2}}{h_{e2}}, \tag{9}$$

where m is defined by

$$I_{12}(0) = \{1 + m\sin(2\pi\nu t + \theta)\}\cos\omega_c t. \tag{10}$$

Thus, the output from the lock-in amplifier is proportional to the square of the field factor $F_{10}(\theta_1, \beta h_1)$, or field intensity.

It should be mentioned here that according to the results of the analysis in the Appendix (see (26)), as the diameter of the coil spring is increased the component $E_{12\phi}(R_0, \theta_2)$ is no longer negligible in comparison with $E_{12\theta}(R_0, \theta_2)$, but it was also proved that $E_{12\phi}(R_0, \theta_2)$ is independent of the length of the helix provided the longitudinal displacement does not change the radius of the helix. It follows that the presence of the component $E_{12\phi}(R_0, \theta_2)$ does not introduce any error in the present scheme of measurements.

Experimental Details

The coil spring used as a probe was arranged horizontally and kept taut between a long nylon thread and a thin but rigid plastic rod which was supported by a vertical plastic stand. The end of the thread was tied to a piston which was moved to and fro by an eccentric wheel which was rotated at 1800 rpm (30 cps) by a small synchronous motor. A shutter was installed on the piston to interrupt periodically a light beam which

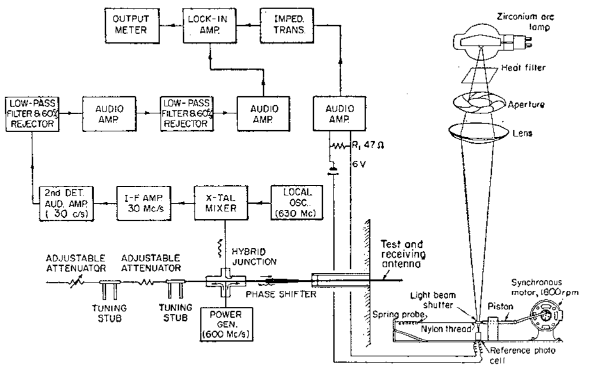

Fig. 2—Block diagram of experimental equipment for measuring electromagnetic field by a coil spring.

illuminated a photocell. The frequency of interruption was the same as the frequency of vibration of the spring. The output from the photocell was used as a reference signal for the lock-in amplifier. Fig. 2 illustrates the complete arrangement. When the frequency of the vibration was too high, the nylon thread pulled the coil spring before the latter had finished a complete cycle; when it was too slow it was difficult to stabilize the high-gain amplifier. A vibration frequency of 30 cps was chosen to suit the stiffness of the available thin steel coil spring. The coil spring was checked for transverse motion of the axis by means of a strobe-light, but no such motion was observed.

It should be noted that any vibrating part in the system which is illuminated by the incident wave gives rise to additional signals to the lock-in amplifier and so leads to errors in the measurement. However, this is true only if vibration is coherent with that of the coil spring. In order to prevent such errors, the wooden frame shown in Fig. 3 was built. A half-wave monopole (at 600 Mc) was horizontally installed on a vertical aluminum image plane on whose edges absorbing material was arranged to reduce reflections. The frame, which supports a horizontal polyfoam sheet on which the coil spring and its driving systems rest, is mechanically free from the ground plane. The zirconium arc-light source is suspended by four wheels which ride on two cylindrical rods which can roll on the horizontal wooden racks. Thus, it is movable in a plane so that the distance between it and the reference photocell is kept constant. In order to make sure that an additional coherent signal due to an unwanted vibration was not received the system was operated with the coil spring absent, and the null signal output from the lock-in amplifier was checked.

The general method of making relative phase measurements as a function of the location of the probe has been fully described elsewhere[9,10,18] and need not be

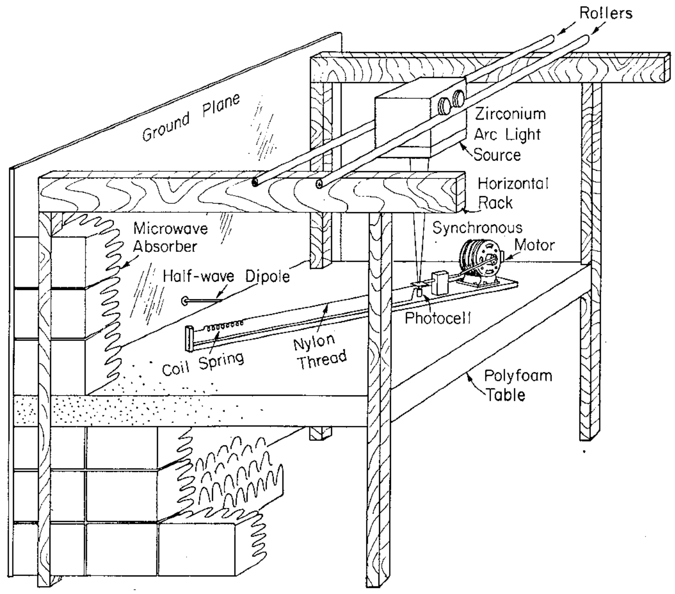

Fig. 3—Construction of the support of the probe and the source of beam of light.

repeated. There are, however, two remarks peculiar to the present setup that should be mentioned. One is the fact that the modulated output from the scatterer [see (8)] is proportional to the square of the field intensity. Hence, the phase measured is twice the actual value plus an arbitrary constant. The other point is that the superheterodyne detection system must be replaced by a square-law detector[9,10,17] which permits a rather simple and easy null method that utilizes the shift of the operating point of the crystal detector by adding the unmodulated reference signal.

Experimental Results

The gains of coil-spring probes with three different dimensions have been tested. The field pattern of a half-wave dipole was also measured with the probe and the results gave a rather satisfactory agreement with theory.

A rough estimate of the over-all amplification gave gains that exceeded 90 db. Two pairs of 60 cps twin-tee band-rejector and low-pass filters were arranged between the stages in order to prevent the saturation of the final tube by the 60 cps pick-up.

The relative outputs of probes with different dimensions were tested by placing the coil spring probe at $\theta_1=90°$, $R_0=\lambda$, $(h/R_0=0.25)$ perpendicular to the ground plane $(\theta_2=90°)$. The relative output from the lock-in amplifier in an arbitrary linear scale is shown in Table I together with the axial length L/λ, diameter D/λ, number of turns n, pitch angle α, magnitude of the mechanical modulation $\Delta h_{e2}/h_{e2}$ and the gain constant K. (Illustration of the quantity K is given below.)

It is seen from the table that the output from the lock-in amplifier grows larger with an increase in the value

[18] J. H. Richmond, "Measurement of time-quadrature components of microwave signals," IRE Trans. on Microwave Theory and Techniques, vol. MTT-3, pp. 13–15; April, 1955.

TABLE I

RELATIVE OUTPUT FROM THE LOCK-IN AMPLIFIER WITH THE DIMENSIONS OF COILS

	Length L/λ	Diameter D/λ	No. of Turns n	Pitch angle a	Displacement $\Delta h_{e_2}/h_{e_2}$	h_{e_2}(in cm) $\times \Delta h_{e_2}$ (in cm)	$K = \frac{\text{output}}{\Delta h_e \cdot h_e}$	Output in Arbitrary Scale
Coil A	0.080	0.006	48	5°30′	0.24	3.8	10.5	39.8
Coil B	0.068	0.008	33	4°40′	0.28	3.2	11.9	38.5
Coil C	0.028	0.006	17	5°00′	0.34	0.7	4.1	2.7

of $(h_{e2} \cdot \Delta h_{e2})$. The signal from the shortest coil C is quite close to the noise level and thus seems to be the lower useful limit of the axial length $(L/\lambda = 0.028)$. From (8) it was found that the amplitude of the signal is proportional to the product $h_{e2} \cdot \Delta h_{e2}$. That is,

$$\Delta I_{12}(0) = K h_{e2} \cdot \Delta h_{e2}. \tag{11}$$

It is interesting to see if the constant K calculated from the experimental results of one coil spring agrees with those of another coil. The value of K for coil A was 10.5 and that for coil B was 11.9, but that for coil C was only 4.1. The values for coils A and B agree rather well, but that for coil C is much smaller than the other. This may be because h_{e2} is so short that the value $\Delta h_{e2}/h_{e2}$ became too large $(\Delta h_{e2}/h_{e2} = 0.34)$ to satisfy the condition

$$\frac{\Delta h_{e2}}{h_{e2}} \ll 1, \tag{12}$$

which was imposed in the derivation of (25).

The choice of the dimensions of the coil spring depends on two quantities: one is $\Delta h_{e2}/h_{e2}$ and the other is $\Delta h_{e2} \cdot h_{e2}$. The values h_{e2} and $\Delta h_{e2}/h_{e2}$ should be selected as small as possible, whereas the quantity $\Delta h_{e2} \cdot h_{e2}$ (which determines the modulated signal level) should not be too low since a detectable signal is required within the range R_0 of interest.

The polar graph of the radiation pattern of a half-wave dipole (quarter-wave monopole over the image plane) measured by the coil spring probe is shown in Fig. 4 along with the theoretical curve. These measurements correspond to one-quarter of the familiar doughnut-shaped pattern. The dipole has its center at $z = 0$ and its end at $z = \pm h_1$. The confocal coordinates k_e and k_h are the reciprocal eccentricities $1/e_e$ and $1/e_h$ of a family of ellipsoids and a family of hyperboloids with foci at $z = \pm h_1$. The field component E_ϵ, which is tangential to the ellipsoid, was measured along the circumference $k_e = 2.0$. The theoretical curve for E_ϵ was calculated including the $1/R^2$ terms by King.[17] The experimental results are in good agreement with theory. In the region very close to the image plane the agreement is less satisfactory. This may be due to the mutual coupling of the probe with its image.

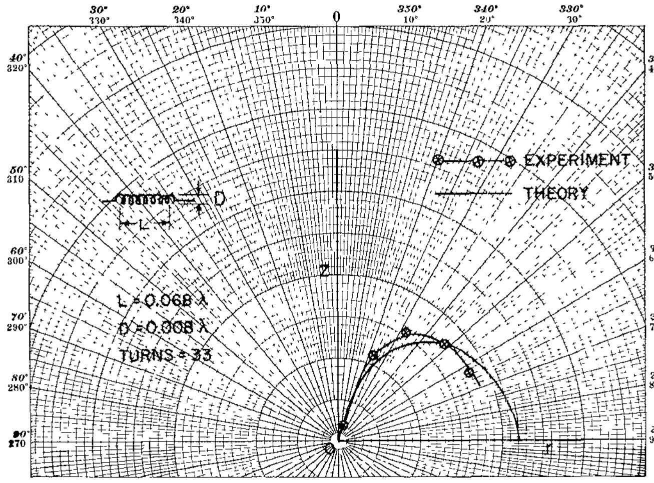

Fig. 4—Polar graph of E_ϵ along the circumference $k_e = 2.0$ on elliptic coordinates measured by a coil spring.

APPENDIX

MODULATION OF EFFECTIVE LENGTH OF A VERY THIN HELICAL ANTENNA DUE TO AXIAL VIBRATION

By comparing the expression for the far-zone radiation field of a very thin helical antenna with that of a short dipole antenna that has a known effective length, the effective length h_e of the helical antenna was obtained.

First, an approximation expression for the radiation field of a very thin helical antenna is derived from the rigorous but rather complicated integrals of Kornhauser.[19] The desired integrals are obtained from the general expression for the far-zone field of a thin wire[20] in terms of the unit radius vector $\hat{R}$ at P, and unit vector $\hat{s}$ along the wire of the helix

$$\overline{E} = \frac{j\omega\mu_0}{4\pi} \frac{1}{R} \int [\hat{R} \times (\hat{R} \times \hat{s})] I_s e^{-j\beta R} ds,$$

where, in the spherical coordinates,

$$\hat{R} \times (\hat{R} \times \hat{s}) = -\hat{\phi}(\hat{s})_\phi + \hat{\theta}(\hat{s})_\theta.$$

[19] E. T. Kornhauser, "Radiation field of helical antennas with sinusoidal current," *J. Appl. Phys.*, vol. 22, pp. 887–891; July, 1951.

[20] R. W. P. King, "Electromagnetic Engineering," McGraw-Hill Book Co., Inc., New York, N. Y., pp. 268–271; 1945.

With reference to the geometric configuration and notations in Fig. 5, it is seen that

$$R = R_0 - (a \tan \alpha \cos \theta)\phi - a \sin \theta \cos \phi$$

and

$$ds = a \sec a \cdot d\phi.$$

With these formulas the following rigorous expression for the field of a helical antenna of arbitrary size may be obtained:

$$E_\phi = -\frac{j\omega a \mu_0}{4\pi} \frac{e^{-j(\omega/c)R}}{R} \left\{ \int_{-\phi_0}^{\phi_0} I \cos \phi e^{j((\omega/c)a \sin \theta) \cos \phi} \cdot e^{j((\omega/c)a \tan \alpha \cos \theta - \beta')\phi} d\phi \right\} \quad (13)$$

$$E_\theta = -\frac{j\omega a \mu_0}{4\pi} \frac{e^{-j(\omega/c)R}}{R} \left\{ \tan \alpha \sin \theta \int_{-\phi_0}^{\phi_0} I e^{j((\omega/c) \sin \theta) \cos \phi} \cdot e^{j((\omega/c)a \tan \alpha \cos \theta - \beta')\phi} d\phi + \cos \theta \int_{-\phi_0}^{\phi_0} \sin \phi \cdot e^{j((\omega/c) \sin \theta) \cos \phi} e^{j((\omega/c)a \tan \alpha \cos \theta - \beta')\phi} d\phi \right\}. \quad (14)$$

These expressions do not involve any approximations; Kornhauser evaluated the integrals by expansion in series. The computation of the fields from his results is rather involved.[19]

If the assumption is made that both the length h_2 and the radius a of the helix is much shorter than the free-space wavelength, the current I along the wire has the triangular amplitude distribution decreasing toward the ends and the same phase along the entire helix. Later the additional restriction is imposed that the length h_2 of the helix is much larger than its radius. Namely the following relations are satisfied:

$$I = I_0 \left[1 - \frac{|\phi|}{\phi_0} \right] e^{-j\beta'\phi} \doteq I_0 \left[1 - \frac{|\phi|}{\phi_0} \right]$$

$$\beta a \sin \theta \ll 1$$

$$\beta a \tan \alpha \cos \theta \ll 1, \quad (15)$$

where

$$\beta = \frac{\omega}{c} = \frac{2\pi}{\lambda}.$$

If these relations are introduced into the integrand of (13) and (14),

$$E_\phi \doteq -\frac{j\omega a \mu_0 I_0}{4\pi} \frac{e^{-j\beta R}}{R} \left\{ \int_{-\phi_0}^{\phi_0} \left[1 - \frac{|\phi|}{\phi_0} \right] \cdot \cos \phi \left[1 + j\beta a \sin \theta \cos \phi \right] \cdot \left[1 + j\beta a \tan \alpha \cos \theta \cdot \phi \right] d\phi \right\} \quad (16)$$

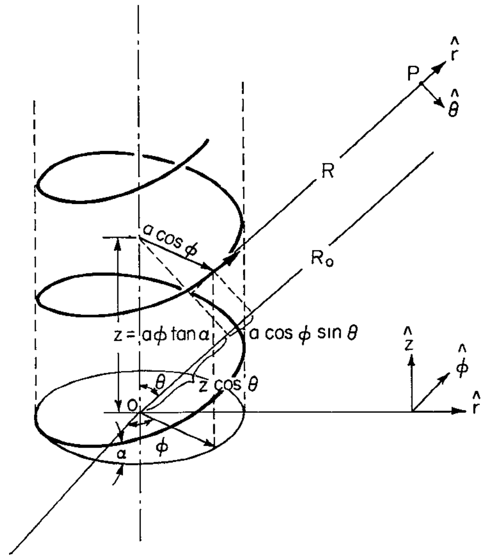

Fig. 5—Geometric configuration of the helix.

$$E_\theta \doteq -\frac{j\omega a \mu_0 I_0}{4\pi} \frac{e^{-j\beta R}}{R} \left\{ [\tan \alpha \sin \theta] \int_{-\phi_0}^{\phi_0} \left[1 - \frac{|\phi|}{\phi_0} \right] \cdot \left[1 + j\beta a \sin \theta \cos \phi \right] \left[1 + j\beta a \tan \alpha \cos \theta \cdot \phi \right] d\phi + \cos \theta \int_{-\phi_0}^{\phi_0} \sin \phi \left[1 - \frac{|\phi|}{\phi_0} \right] \left[1 + j\beta a \sin \theta \cos \phi \right] \cdot \left[1 + j\beta a \tan \alpha \cos \theta \cdot \phi \right] d\phi \right\}. \quad (17)$$

The evaluation of these integrals is straightforward; the results are

$$E_\phi = 2 \frac{E_0}{\phi_0} \cdot \left\{ 1 - \cos \phi_0 + jp \left[\frac{\phi_0^2}{4} + \frac{1}{8}(1 - \cos 2\phi_0) \right] \right\} \quad (18)$$

$$E_\theta = 2 \frac{E_0}{\phi_0} \frac{q}{a\beta} \left\{ \frac{\phi_0^2}{2} \tan \theta + \frac{p}{8} a\beta \cdot \cos \theta (\phi_0 \sin 2\phi_0 + \cos 2\phi_0 - 1) + j[p \tan \theta (1 - \cos \phi_0) - a\beta \cos \theta (2 \cos \phi_0 - 2 + \phi_0 \sin \phi_0)] \right\}, \quad (19)$$

where

$$p = \beta a \sin \theta$$

$$q = \beta a \tan \alpha \cos \theta$$

$$E_0 = -j \frac{\omega a \mu_0 I_0}{4\pi} \frac{e^{-j\beta R}}{R} = -j\eta \frac{a I_0}{2\lambda}$$

$$\eta = \sqrt{\frac{\mu_0}{\epsilon_0}}. \quad (20)$$

Note here that E_ϕ is independent of $q(\alpha)$ and hence is not modulated by the axial vibration of the helix. (The only quantity associated with an axial vibration is the pitch angle α if the displacement is limited to a range for which the radius a is constant.)

Assume for simplicity that the helix has an integral number of turns n and thus $\phi_0 = n\pi$.

$$E_\phi = 2E_0\left\{\frac{1-(-1)^n}{n\pi} + j\frac{pn\pi}{4}\right\} \tag{21}$$

$$E_\theta = E_0\frac{q}{\alpha\beta}\frac{2}{n\pi}\left\{\frac{(n\pi)^2}{2}\tan\theta + j[1-(1)^n] \cdot [p\tan\theta + 2\alpha\beta\cos\theta]\right\}. \tag{22}$$

The leading term of (22) is reduced to simpler form with (20). Thus, for a helical antenna,

$$E_\theta = -j\eta\frac{I_0}{2\lambda R}e^{-j\beta R}\sin\theta(n\pi a\tan\alpha). \tag{23}$$

It is interesting to compare this result with that of a short antenna with a constant current along the length h[21]

$$E_\theta = -j\eta\frac{I_0}{2\lambda R}e^{-j\beta R}\sin\theta\cdot h. \tag{24}$$

It follows that the effective length h_{e2} of a thin helical antenna is

$$h_{e2} = n\pi a\tan\alpha. \tag{25}$$

[21] H. H. Skilling, "Fundamental of Electric Waves," John Wiley and Sons, Inc., New York, N. Y., 2nd ed., p. 168; 1948.

Referring to Fig. 5, it is seen that (25) is nothing but one-half of the physical axial length of a thin helical antenna.

Next the modulation of E_ϕ and E_θ due to an axial vibration is obtained. For E_ϕ, it is obvious from (18) that

$$\Delta E_\phi = \frac{dE_\phi}{dh(\alpha)}\Delta h = 0. \tag{26}$$

Since (25) demonstrated that the effective length h_e of the helix with a triangular distribution of current is the same as one-half of the physical length h_2 subject to the assumptions in (15) the amplitude ΔE_θ of the modulation is

$$\Delta E_\theta = -j\eta\frac{I_0}{4\lambda R}e^{-j\beta R}\sin\theta\cdot\Delta h_2 = E_\theta\frac{\Delta h_2}{h_2}\cdot \tag{27}$$

A slightly more accurate approximate expression may be obtained by inserting (20) and (22) into

$$\Delta E_\theta = \frac{dE_\theta}{dh_2}\Delta h_2 = \frac{dE_\theta}{dq}\frac{dq}{d\alpha}\frac{d\alpha}{dh_2}\Delta h_2. \tag{28}$$

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